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How to Cut the Seigniorage Cake into Fair Shares in an Enlarged EMU

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Abstract: Beginning in 2008 seigniorage from the issuing of euro notes will be redistributed by the European Central Bank to the euro countries in accordance with a key based on the respective GDP and population proportions of the euro countries. Applying this distribution formula to the new EU countries from Central and Eastern Europe seems to give these countries a net benefit compared with the seigniorage they bring in, i.e. their share of currency in circulation. However, as argued in this paper currency demand in the new member countries is expected to increase relative to the present group of euro countries and, hence, it is doubtful whether an unintended redistribution of seigniorage will take place in the long run. It is also argued in this paper that the individual countries' contribution to seigniorage may differ from what the countries give up in national currency seigniorage because of international seigniorage, institutional and behavioural changes, and changes in monetary policy after enlargement.

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1 Introduction

On May 1, 2004 the European Union expanded to include ten new member states. All new member states have the obligation to seek membership in the Economic and Monetary Union (EMU) after fulfilling the Maastricht criteria. Once EMU membership is a reality the new member countries of the euro group have delegated their monetary autonomy to the European Central Bank (ECB). The ECB is also the core institution for collection and administration of seigniorage, or profits, from the issue of euro currency. These amounts of seigniorage are, in the first round, collected by the ECB, but afterwards redistributed to the national central banks (NCB's) in accordance with a specific allocation key.

During the last ten years the accession process has inspired a vast amount of analysis about the effects of the widening of the Single European Market. Likewise, the implications for efficiency and macroeconomic stability of EMU enlargement have received significant attention from researchers. By comparison the distributional issue of seigniorage in an enlarged EMU has only been addressed in few analyses. The purpose of this paper is to add to the relatively scanty literature by investigating whether the individual member countries of the EMU will receive their 'fair share' of the seigniorage from the ECB. In particular, we focus on the anticipated distribution of ECB seigniorage to the ten new, mostly east European, EU member countries when they, as expected, enter the EMU before the end of this decade.

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The legal framework for distribution of seigniorage does not point to any objective of transfer of seigniorage in favor of relatively poor EMU member countries - i.e. there is no subsidy transfer motive. "Fair shares" is, therefore, in the following defined as a distribution of seigniorage in accordance with the amounts of seigniorage that the individual countries bring into the common pool for redistribution. In other words, if wealth redistribution occurs it does so by default, and not by design.

The overall provisions for transfer of seigniorage from the ECB to the member countries of the euro zone are outlined in the protocol "Statute of the European System of Central Banks and the ECB" which is related to the Maastricht Treaty. After the current transitional period ends in 2008 the protocol prescribes that the transfer of seigniorage to the individual euro zone countries should reflect the individual countries' relative size in the euro area. Here relative size is measured by an average of the country's proportion of GDP and population.

However, during the present transitional period, until 2008, pooled seigniorage is allocated to the individual euro zone countries according to a modification of the formula: partly according to the above distribution rule, and partly according to the countries' respective shares of (national currency) bank notes prior to 2002, i.e. before the adoption of the euro as common currency. (see Scheller 2004, p. 119).

The distributional issue of EMU seigniorage has previously been addressed in analysis of Sinn and Feist (1997, 2000), and, specifically related to the next enlargement of the EMU, by Feist (2001). They have argued that post-euro seigniorage distribution should be guided by a country's pre-euro 'historic seigniorage wealth', if an unintended redistribution of wealth should be avoided. Here historic seigniorage wealth is defined as the stock of currency in circulation prior to the adoption of the euro. Specifically, Feist (2001) argues that the new member countries from Central and Eastern Europe would benefit substantially by EMU membership, as these countries' contribution to pooled seigniorage is expected to be significantly smaller than what they are to receive according to the ECB's redistribution formula. By giving weight to the magnitude of (pre-euro) currency in circulation the transitional allocation rule for seigniorage, in part, reflects the views of Sinn and Feist.

In this paper it is argued that the redistribution formula, to some extent, has a theoretical foundation in the demand theory for money, and that the derived key for distribution of pooled seigniorage can be regarded as an approximation of the proportions of currency in circulation.

Although historic seigniorage wealth is, admittedly, low for new member countries from Central and Eastern Europe, it is unlikely that this will persist in the longer term, after these countries adopt the euro. As discussed in this paper the demand for currency is, for a number of reasons, expected to increase in the Central and Eastern European countries, relative to the current EMU members. Hence, in the coming years their share of contributions to the common seigniorage pool will grow.

The paper is organized as follows. Section 2 sets up a simple formal framework for the formation of seigniorage in a monetary union for a group of countries. Following Sinn and Feist, it is argued that seigniorage may be perceived as social wealth i.e. a stock variable which equals the net present value of the flow of central bank profits from issuing currency. Section 3 describes the ECB formula for distribution of seigniorage, and discusses the economic rationale for the weights given by the formula. Section 4 compares the shares of stock of currency of the new EU-countries with the shares in accordance to the distribution key. The latter shows the actual schemes of shares after the enlargement of the EU as of May 1, 2004. Based on these figures the new member countries from Central and Eastern Europe seem to benefit strongly from the ECB key. However, this conclusion is based on historic seigniorage from 2002, i.e. several years before membership of the EMU will take place. Section 5 analyses the problems of using ante EMU stock of currency as an indicator for post EMU demand for currency. It concludes that this apparent large overpayment to new member countries may not materialize when membership in the EMU takes place. Section 6 concludes.

2 A model for seigniorage in a monetary union

This section deals with some basic concepts of seigniorage in a monetary union. The central bank's seigniorage profit derives from the use of the banks' exclusive right to issue base money. This allows the central bank to establish liabilities with no, or very little, interest payments. Through this profitable activity the central bank contributes to financing the public sector's expenditures; i.e. seigniorage is an implicit tax. In the following we only look at seigniorage generated from the central banks issue of currency. This is by far the largest amount of seigniorage and also the most controversial in the distributional discussion within EMU, as it is only this part of seigniorage that is socialized by the ECB, and subsequently redistributed.

The central bank issues currency to satisfy the demand for this asset. Primarily the demand for currency comes from persons, firms and institutions in the issuing country to satisfy their need for means of payment and store of value, but some currencies also enjoy the advantage to be demanded by external agents. However, in the basic model presented below currency is assumed to be used only internally.

A simple specification of money demand, see e.g. Fair (1987), assumes that the real money demand per capita depends on real GDP per capita, the nominal interest rate, i , and specific factors, c . Applying such specification also for the stock of currency in circulation, B , we have for a given monetary area;

$$B/PN = c y^\alpha i^{-\beta}; \quad 0 < \alpha < 1, \quad \beta > 0 \quad (1)$$

where P stands for the price level, N for the number of inhabitants, α and $-\beta$ for elasticities with respect to per capita income and nominal interest rate, respectively.

The transfer of seigniorage from the central bank to the government may, in principle, proceed in accordance with two alternative procedures. One procedure is to transfer directly the growth of the stock of currency to the government. Alternatively, the central bank can use the growth of the stock of currency to buy interest bearing assets and transfer the return from these assets to the government. Corresponding to the two procedures seigniorage may be perceived as a cash flow S_C or as a flow of earnings S_E i.e.:

$$S_C = dB/dt \quad (2)$$

or:

$$S_E = iB \quad (3)$$

The current flows of seigniorage (2) and (3) are, in general, different. However, in a multi- period analysis of seigniorage an ex ante assessment of present value of earnings seigniorage and cash flow seigniorage are equal, when the consolidated debt of the central bank and the government is taken into account. To illustrate this let us assume that the economy is in steady state growth with a population growth rate n , a real rate of growth of per capita GDP, g , and a real interest rate, r , all constants over time. Furthermore, let us assume that the rate of growth of currency, μ , and the monetary multiplier are constant, so that the rate of growth of currency translates into a similar rate of growth of money supply. Applying the quantity theory of money and the Fisher equation the rate of inflation, p , and the nominal interest rate, i , will then also be constants given by:

$$p = (\mu - n - \alpha g) \quad (4)$$

and:

$$\begin{aligned} i &= (r + p) \\ &= (r + \mu - n - \alpha g) \end{aligned} \quad (5)$$

Using these assumptions the demand for currency is given by:

$$\begin{aligned} B &= cP_0N_0y_0^\alpha (r + \mu - n - \alpha g)^{-\beta} e^{\mu t} \\ &= B_0 e^{\mu t} \end{aligned} \quad (6)$$

where P_0 , N_0 and y_0 is the present price level, size of population and present per capita income, respectively. B_0 is the present level of demand for currency equal to $cP_0N_0y_0^\alpha (r + \mu - n - \alpha g)^{-\beta}$. Note that the equality between

B_0 and $cP_0N_0y_0^\alpha (r + \mu - n - \alpha g)^{-\beta}$ for alternative values of the growth of money supply μ is secured by an endogenously determined P_0 , as B_0 is policy determined. i.e. a larger rate of growth of money supply μ corresponds to a higher nominal interest rate i and a higher price level P_0 in monetary equilibrium. In other words, an alternative higher rate of growth of currency supply μ corresponds to a lower present real value of stock of currency B_0/P_0 .

If the public sector only appropriates earnings seigniorage the net debt of the consolidated balance for central bank and the public sector is unaffected by the use of seigniorage. Hence, present value of earnings seigniorage at time 0, WS_E , for the flow of earnings from time 0 to time T is given by:

$$\begin{aligned} WS_E &= \int_0^T iBe^{-it} dt \\ &= B_0 \left[1 + \frac{\mu}{(r-n-\alpha g)} \right] (1 - e^{-(r-n-\alpha g)T}) \end{aligned} \quad (7)$$

Assuming that $r > n + \alpha g$ (which is in accordance with standard neoclassical growth theory, see e.g. Romer (1996)), WS_E stabilizes for $T \rightarrow \infty$ at the long term level:

$$WS_E^* = B_0 \left[1 + \frac{\mu}{(r-n-\alpha g)} \right] \quad (8)$$

If cash flow seigniorage is appropriated the consolidated (public sector and central bank) net debt to the private sector is B_0 at time 0 and $B_0 e^{\mu T}$ at time T. Hence, present value of cash flow seigniorage at time 0 is:

$$WS_C = B_0 + \left(\int_0^T dB/dt e^{-it} dt \right) - B_0 e^{-(\mu-i)T} \quad (9)$$

From (6) we have $dB/dt = B_0 \mu e^{\mu t}$

Inserting this in (9) and using (4) and (5) gives:

$$\begin{aligned} WS_C &= B_0 \left[1 + \mu \int_0^T e^{-(r-n-\alpha g)t} dt - e^{-(r-n-\alpha g)T} \right] \\ &= WS_E \end{aligned} \quad (10)$$

i.e. a forward looking assessment of present value of cash flow seigniorage is equivalent to present value of earnings seigniorage.

If expectations are not fulfilled windfall gains and losses might appear, and this may cause differences between *ex post* assessments of earnings and cash flow seigniorage wealth. Most obviously, unanticipated inflation offers more cash flow seigniorage to the government compared with earnings seigniorage, in the case where the cash flow seigniorage has been used for public expenditures and hence, the public sector, including the central bank, has become indebted to the private sector. The real debt burden for the public sector will then be partly written off by unanticipated inflation. A similar debt relief will not appear if the government only uses the earnings seigniorage. In this case seigniorage will not be associated with an increase in nominal net debt of the public sector.

In a stationary non-inflationary economy ($g = n = 0$ and $\mu = p = 0$) the long-term level (8) equals the initial stock of currency B_0 , which represents what Sinn and Feist (1997) termed "historic" seigniorage. In the more general case of growth and

inflation total seigniorage wealth given by (8) consists of historic seigniorage B_0 plus a term which depends on the rates of growth of stock of currency (money), population, per capita income, respectively, and the elasticity of currency demand with respect to per capita income.

The analysis above has been related to one country only. We will now look at seigniorage for different countries, each of which has its own currency. We disregard international seigniorage i.e. that some countries' currencies are demanded by other countries. To simplify matters, the purchasing power parity theory is assumed to hold. In this case seigniorage wealth can be compared between different countries by real value of seigniorage. Present real value of seigniorage wealth for an individual country RWS^* for an infinite time horizon is given by (8) using the expression for B_0 from (6).

$$\begin{aligned} RWS^* &= B_0/P_0 \left[1 + \frac{\mu}{(r - n - \alpha g)} \right] \\ &= \frac{cN_0 y_0^\alpha (r + \mu - n - \alpha g)^{-\beta-1}}{(r + n - \alpha g)} \end{aligned} \quad (11)$$

For countries, which pursue an independent monetary policy, the present real value of seigniorage wealth depends on the monetary policy specified by the rate of growth of money supply μ . However, the implication for RWS^* of an alternative higher value of μ depends on the elasticity of currency demand with respect to the nominal interest rate. The first-order derivative of RWS^* with respect to μ is positive (negative) for $-\beta$ less (larger) than 1. The real value of historic seigniorage B_0/P_0 decreases due to a higher nominal interest rate, but the second term in the first line of (11) increases, reflecting the direct effect of the faster rate of growth of money supply¹. The real value of historic seigniorage is therefore not linked in any simple way to the present real value of seigniorage wealth.

This conclusion also holds if the countries pursue the same monetary policy, defined as a monetary policy which gives the same inflation rate i.e. $\mu = (p_0 + n + \alpha g)$ implying an inflation rate at p_0 for all countries. Inserting this assumption into (11) gives:

$$\begin{aligned} RWS_{p \text{ const.}}^* &= B_0/P_0 \left[\frac{(r + p_0)}{(r - n - \alpha g)} \right] \\ &= \frac{cN_0 y_0^\alpha (r + p_0)^{-\beta-1}}{(r - n - \alpha g)} \end{aligned} \quad (11a)$$

¹ RWS^* might be a non monotonic function of μ in more general non-iso-elastic specifications of the currency demand function and in such cases a finite growth rate in money supply which maximizes RWS^* might exist. In the literature on the 'optimum mix' of taxes on money and other tax sources the relationship between receipts of seigniorage for various rates of growth of money supply plays a crucial role, see e.g. Easterly et al. (1995) for an analysis of this issue.

For countries pursuing the same monetary policy the present real value of seigniorage wealth thus varies positively with the rate of growth of population, and the rate of growth of real per capita income, for a given real value of historic seigniorage. Hence, there is no one to one link between real value of historic seigniorage and present real value of seigniorage wealth, neither in the case where the countries pursue different monetary policy nor in the case where monetary policy is harmonized. In the special case of monetary policy harmonization the fast growing economies will have a larger present real value of seigniorage wealth relative to real value of historic seigniorage compared with low growth countries.

What countries give up in seigniorage wealth by entering a monetary union is described by present real value of seigniorage wealth given by (11) or (11a). Several years before membership in the monetary union the potential member country may still have its monetary policy autonomy. In this case the present real value of seigniorage wealth, which the country gives up, is then given by (11). When the date for membership approaches potential member countries have a very strong incentive to let their monetary policy converge to the monetary policy of the future partner countries in order not to expose their economies (and future partner economies) to a monetary shock. Hence, at the doorstep to the monetary union (11a) represents what the country gives up of real seigniorage wealth².

For countries with their own currency historic seigniorage wealth is directly measurable. However, the relevant stock variable for seigniorage wealth is present real value of seigniorage wealth, an assessment of which relies, as appears from (11) and (11a), on assumptions about expected future real interest rates, monetary policy, and economic growth. These expectations may turn out to be wrong, leaving ex post present real value of seigniorage different from the ex ante value. The following analysis of seigniorage in a monetary union, therefore, turns to a flow assessment of seigniorage contributions from individual countries in the specific year.

In a monetary union, where the common currency has no international role, each member country contributes to union earnings seigniorage in proportion to the stock of currency in circulation in the domain of the country. Let us assume that the demand function for the individual member countries has the form specified by (1), and to simplify, assume furthermore that the parameters are the same for all member countries apart from c , which captures country specific factors other than per capita income y and interest rate i . The currency demand function for member country j is thus:

$$B_j / P_j N_j = c_j y_j^\alpha i_j^{-\beta}; 0 < \alpha < 1, \beta > 0 \quad (12)$$

Supply of currency relates to the entire monetary union, and, at least for relatively small member countries, the demand, given by (12), is satisfied by the common stock of currency; i.e. for the individual member country supply of currency is purely demand driven. Hence the rate of growth of currency is now endogenously determined, depending, among other things, on the exogenously given rate of inflation.

² In the EMU-case monetary policy convergence is a 'must' included in the Maastricht criteria especially about convergence of rate of inflation and interest rate.

The nominal interest rate is assumed to be equalized across countries in the monetary union and hence, demand for currency by country j relative to total union demand may be written as:

$$B_j = c_j P_j N_j y_j^\alpha \quad (13)$$

where the union variables B_U, c_U, P_U, N_U, y_U are normalized to 1.

We assume that the monetary union is also complemented by free mobility of goods and services, but because of non-tradable goods and services the price levels might differ. However, in accordance with the reasoning behind the Balassa-Samuelson effect the relative price level varies positively with the relative real per capita income. For convenience we specify this relationship by the function:

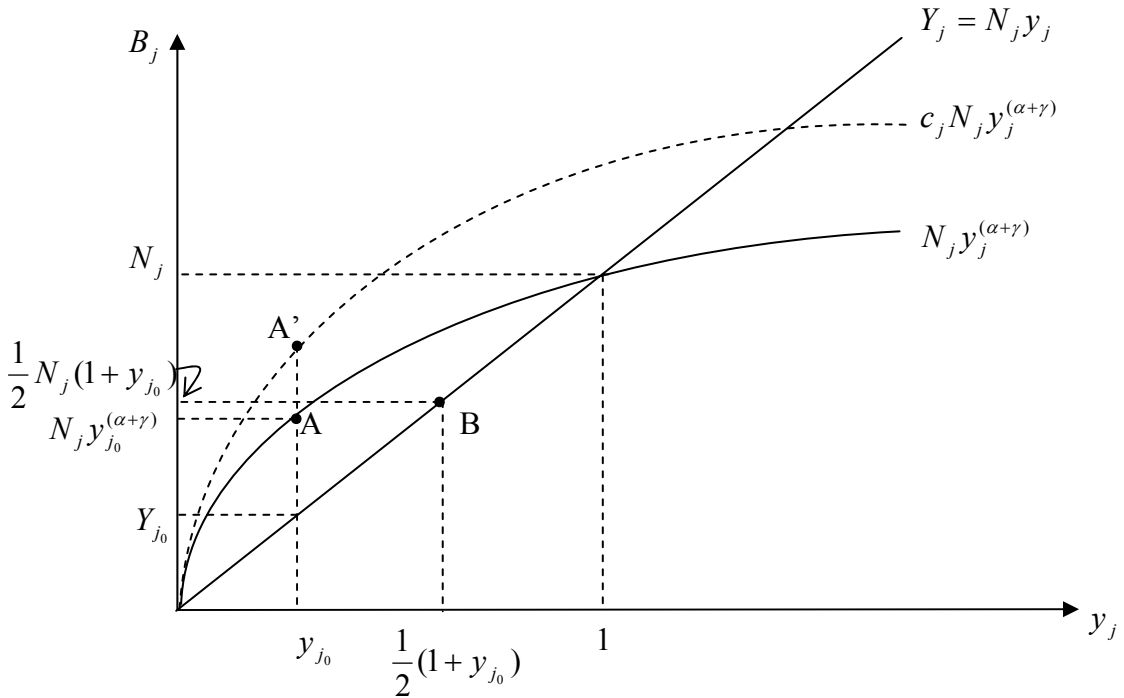
$$P_j = y_j^\gamma; \quad 0 < \gamma < 1 \quad (14)$$

Inserting (14) into (13) gives the final expression for share of currency for country j :

$$B_j = c_j N_j y_j^{(\alpha+\gamma)} \quad (15)$$

where it is assumed, that $(\alpha + \gamma) < 1$.

Figure 1: Share of currency and relative per capita GDP



The relative demand for currency, B_j , for a euro country j , is illustrated as a function of per capita income, y , in Figure 1. The solidly drawn curve illustrates the relative demand function in the case where no country specific factors, apart from per capita income, influences the currency demand, i.e. $c_j = 1$. In this case the country's share of currency in circulation equals the population share if its per capita income is at par with the average of the union. If per capita income of country j is below average the relative currency demand of the country is less than its share of population but larger than its share of GDP (the latter is indicated by the line $N_j y_j$ in Figure 1). This discrepancy between share of currency and share of GDP, $(B_j - N_j y_j)$, decreases for $(\alpha + \gamma) \rightarrow 1$ for given y_j . In the special case, where, $(\alpha + \gamma) = 1$, the share of currency equals the share of GDP for all y_j . To express this alternatively, if the income elasticity of currency demand α is large, and/or the parameter γ for the Balassa-Samuelson effect is large, the country's share of currency is close to the country's share of GDP.

However, country specific factors might influence the position of the relative demand function. If, for example, the country has a more cash intensive tradition ($c_j > 1$), this specificity will be reflected in a relative high share of currency as illustrated by the dashed curve in Figure 1.

The above analysis complicates if the common currency is demanded internationally for private use or as international reserves in foreign central banks. The monetary union will, in this case, earn international seigniorage, in addition to the seigniorage earned in its own domain. This extra seigniorage cannot, in any logical way, be assigned to parts of the monetary union. Some of the former national currencies, prior to the formation of the monetary union, may have earned international seigniorage because of their reputation for monetary stability. This has clearly been the case for the German mark, at the time the euro replaced the German mark. Such a country gives up its capacity to earn international seigniorage by entering a monetary union. But the common currency may, at least in the beginning of its existence, benefit from the reputation of the former, but now liquidated, currencies. This was also what happened in the case of EMU, where the provisions for the new central bank deliberately mirrored the main provisions for the German central bank, in order to inherit its reputation for monetary stability. However, in the longer term the reputation of the new currency will depend on its record of stability, and none of the participating countries can claim special responsibility for that.

3 The ECB formula for redistribution of seigniorage

Given that no country in the euro zone can claim special rights to international seigniorage the basic principle for distribution of seigniorage in EMU is that not only domestic but also internationally generated seigniorage in the EMU should be distributed as close as possible into shares, which reflect domestic relative currency demand. However, as soon as the common currency is introduced national data of currency demand eludes a direct measurement. The demand for currency of the individual member countries can only be assessed indirectly, and this is what the ECB formula intends to do.

The total amount of gross earnings seigniorage from currency in the euro area, i.e. seigniorage before administrative costs and costs for printing money, equals the calculated interest payment on the stock of currency. The ECB monetary strategy aims for price stability, defined as up to 2 percent yearly price increases in the medium term. This corresponds to an interest rate of about 6 percent in the Fisher-equation, assuming a real interest rate of about 4 percent. These monetary figures are not far from the present situation. Euro currency in circulation has, in recent years, amounted to about 5 to 6 percent of GDP³. The earnings seigniorage from currency thus makes up about 0.3 to 0.4 percent of GDP⁴.

The legal framework for the institutions in the EMU is given in the Protocol on the ‘Statute of the European System of Central Banks and of the European Central Bank’, and this document also provide the rules for the ECB formula for redistribution of seigniorage⁵. The distributional issue of seigniorage in the EMU may be divided into two separate questions: how does the ECB obtain its seigniorage in the first place, and how is this seigniorage subsequently redistributed?

The answer to the first question is given by Article 32 of the Protocol. The relevant provisions are a consequence of the procedure for the substitution of the national currency with euro in the initial phase of the EMU. The national central banks (NCBs) of the euro system play a pivotal role as the prime source of earnings of seigniorage. The issuing of euro notes takes place on a joint basis between the ECB and the NCB’s. The ECB issues only 8 percent of the total value of bank notes and this contributes directly to the ECB seigniorage. The remaining 92 percent is put into circulation by the NCB’s. When national currencies were phased out the ECB delivered the euro currency to the participating NCBs, for the claim on a calculated income from assets held against euro notes. The calculated income for the individual NCB is based on specified short-term interest rates in the Eurosystem. This amount is transferred from the NCB to the ECB as a yield on the stock of euro the NCB has put into circulation. These direct and indirect seigniorage income flows contribute to the net profit of the ECB, and make up the basis for redistribution to the NCB’s. The net profit is also influenced by income from other sources, various costs and windfall gains and losses on assets and liabilities. Especially important are exchange and interest rate variations, which can have a big impact on the ECB’s financial result in a particular year⁶.

The answer to the second question on the redistribution of seigniorage follows from Article 33 in the Protocol. The NCBs are formally shareholders of the ECB and the overall principle is to redistribute seigniorage in proportion to the NCBs shareholder position of the ECB. To be more specific, the redistribution will be in accordance with the following three provisions. Firstly, only the euro area-NCBs are recipients for

³ See ECB (2004) Euro Area Statistics.

⁴ A similar simple calculation of earnings seigniorage for the EMU has been made by Gros (1993). He assesses the seigniorage revenue to 0.56 percent of GDP using an interest rate at 7 percent and a share of non-interest bearing primary money (currency plus reserves) at 8 percent of GDP.

⁵ A comprehensive description of the monetary institutions in the EMU is given in Scheller (2004). The following section draws from this source.

⁶ The ECB incurred a net loss in 2003 mainly because of the depreciation of the US dollar (Scheller, 2004).

money transfer of seigniorage. The NCBs, which do not participate in the euro as common currency, are neither entitled to have any part in the money transfer of seigniorage from the ECB, nor liable for losses of the ECB. Secondly, up to 20% of the seigniorage may be transferred to a reserve fund, belonging to the ECB, before redistribution takes place. This reserve fund can be used to cover future losses by the ECB. Thirdly, the shareholder approach for redistribution of seigniorage will start to be applied fully from 2008. In the transitional period until 2008 the principles for redistribution of seigniorage will also take into account the stock of (national) currency in circulation in the individual euro countries in the years 1999 to 2001, i.e. before the introduction of euro currency. The transitional period aims to smooth the transfer of seigniorage from previous levels based on stock of currency in circulation to the amounts based on shareholder positions. However, the new EU member countries from Central and Eastern Europe are not expected to enter the EMU, before 2006, at the earliest. This means that provisions for redistribution of seigniorage according to the transitional rule have only little relevance for these countries.

For redistribution of seigniorage the shareholder position of the NCBs in the euro area is of crucial importance. As not all NCBs in the EU belong to euro-zone countries, Article 28 distinguishes between the NCBs' subscribed capital and paid-in capital. The NCBs of all EU countries hold subscribed capital in the ECB. The NCBs that participate in the euro-zone fully pay in their subscribed capital, while non-euro area NCBs only pay in 7 percent of their subscribed capital. The total amount of subscribed capital after the EU enlargement of May 1, 2004 is about 5.6 billion euro, and the paid in capital about 4.0 billion euro. These amounts are very small relative to the total stock of currency in circulation in the present EU, which at the end of 2002 consisted of about 422 billion euro (see Table 3). The NCBs' shareholdings are therefore only relevant in relative terms i.e. for distribution of seigniorage.

Table 1 reports the recently adopted capital structure of the ECB and the key for redistribution of seigniorage after the widening of the EU from May 1, 2004. The logic behind the reduced paid-in capital for the non-euro countries is that these countries do not get any transfer of seigniorage, but benefit from some tasks performed by the ECB for the non-euro area.

The share of subscribed capital assigned to a NCB is a simple average of the respective country's share of population and share of GDP. The calculated weights are kept constant for five years, and are then adjusted in accordance with the changes in shares of populations and GDPs during this period. To make the specific average of the two shares, the shares of population refer to the year, two years before the five year period, to which the weights apply. To smooth for business cycles the share of GDP for the individual country is calculated as a five-year average of shares of GDP. The latest year in this average is the one two years before the key goes into force. The weights of the actual key put into force on May 1, 2004 are based on shares of population for 2002 and an average of the individual country shares of GDP for the years 1998-2002.

Table 1: Capital keys of the ECB, May 1, 2004.

	Share subscribed capital	Share paid in capital percent	Redistribution key
Belgium	2.55	3.47	3.57
Germany	21.14	28.76	29.57
Greece	1.90	2.58	2.65
Spain	7.78	10.58	10.88
France	14.87	20.24	20.80
Ireland	0.92	1.25	1.29
Italy	13.05	17.76	18.26
Luxembourg	0.16	0.21	0.22
The Netherlands	4.00	5.44	5.59
Austria	2.08	2.83	2.91
Portugal	1.77	2.40	2.47
Finland	1.29	1.75	1.80
<i>Subtotal for euro area NCBs</i>	<i>71.49</i>	<i>97.28</i>	<i>100.00</i>
Denmark	1.57	0.15	
Sweden	2.41	0.23	
U.K.	14.38	1.37	
Poland	5.14	0.49	
Czech Rep.	1.46	0.14	
Slovak Rep.	0.71	0.07	
Hungary	1.39	0.13	
Slovenia	0.33	0.03	
Estonia	0.18	0.02	
Latvia	0.30	0.03	
Lithuania	0.44	0.04	
Cyprus	0.13	0.01	
Malta	0.06	0.01	
<i>Subtotal for non-euro area NCBs</i>	<i>28.51</i>	<i>2.72</i>	
Total	100.00	100.00	
Total amount of capital (billion of Euro)	5.60		

Note: The shares of paid-in capital is calculated by reducing the subscribed capital for the non-euro zone members Denmark, Sweden and UK to 7% of its value and re-scaling. Similarly, the redistribution key is calculated by re-scaling the subscribed (or paid-in) capital for the euro countries only. The figures for subscribed and paid-in capital are defined in four decimals but rounded to two decimals only in the Table.

Source: Scheller (2004).

The formula for distribution in the Protocol may be seen as an approximation of the shares of currency indicated by (15) for $c_j = 1$. This is apparent from Figure 1, which allows for a graphical exposition of the shares given by the formula. The share specified by the Protocol for country j is given by:

$$ECB - share \ of \ country \ j = (N_j + Y_j) / 2 = N_j(1 + y_j) / 2 \quad (16)$$

where Y_j stands for country j 's share of GDP. For a country j with a per capita income y_j relative demand for currency is indicated in Figure 1 by the vertical distance to point A on the demand function. The line $Y_j = N_j y_j$ illustrates relative GDP for various y_j but fixed N_j . The arithmetic mean of the GDP share and population share thus appears as the vertical distance to the point B. In the illustrated case the difference between the two vertical distances is small i.e. the formula approximates the relative demand of currency well.

A closer analysis of the approximation reveals that the sign of the approximation error, i.e. the difference between the share given by the ECB-formula and the share of currency given by the demand function for $c=1$ is ambiguous. The ratio, R , between the ECB-share and the currency demand may be termed the overpayment ratio, as it illustrates what a country gets out of seigniorage relative to what it brings in. In the absence of country specific factors i.e. $c = 1$ the overpayment ratio, R , is calculated from (15) and (16) which gives:

$$R = (1 + y) / 2y^{(\alpha + \gamma)} \quad (17)$$

where the subscript indicating the specific country j is omitted.

Differentiating R with respect to y gives:

$$dR / dy = [1 - (\alpha + \gamma) - (\alpha + \gamma) / y] / 2y^{(\alpha + \gamma)}$$

which is negative for:

$$y < (\alpha + \gamma) / (1 - (\alpha + \gamma)) \quad (18)$$

This condition is fulfilled for poor countries ($y < 1$) if $\frac{1}{2} \leq (\alpha + \gamma) \leq 1$, which is in accordance with most empirical evidence about elasticities of money demand and even in the case $(\alpha + \gamma) < \frac{1}{2}$ relation (18) may be fulfilled for very poor countries.

4. The ECB-key and historic seigniorage

At the time where the monetary union is established, or when new countries adopt the euro, the conversion of national currencies to the euro reveals information about the demand for currency on a country basis; i.e. when the entrance door to EMU is passed historic seigniorage will be revealed for the last time. As suggested by Sinn and Feist (1997) and Feist (2001) this gives an opportunity to assess what the countries

contribute to the pooled seigniorage. In the analysis of Feist (2001) the ECB-key for an enlarged EMU is calculated and compared with the shares of currency for 1998. From this comparison it is concluded that the new member countries from Central and Eastern Europe will receive substantially more seigniorage than the seigniorage they bring in. The calculation is based on data from 1998 for all 25 member countries of the EU plus Bulgaria and Romania.

In the following a similar assessment is made for a slightly different group of perceived future EMU-countries, based on the recently introduced key for subscribed capital, reported in Table 1. The working assumption for the following analysis is that all ten new EU member countries adopt the Euro before 2008⁷. In other words, it is assumed that of the twenty five EU members, twenty two will be members of the Euro-zone. Only the U.K., Sweden and Denmark are assumed to stay outside the Euro-zone. This assumption is justified by reference to the observed eagerness of the ten new entrants to join alliances that ties them politically and economically to Western Europe and the western hemisphere in general. In addition, governments, vulnerable to coalition politicking, as is often the case in Central and Eastern Europe, will find comfort in being able to blame the ECB for monetary policy that is unpopular at home.

On the other hand, excluding the U.K., Denmark and Sweden from the hypothetical enlarged Euro-zone is based, in the case of U.K. and Denmark, on their contractual right to stay outside the Euro-zone indefinitely⁸, and the absence of formal expression of a wish to join. While the contractual arrangement with Sweden facilitates, in principle, a fast entry into the Euro-zone, Sweden so far has not expressed its intention to exercise this option. Our hypothetical enlarged Euro-zone includes the present 12 member countries of the euro zone plus the new EU countries: *Poland, Czech Republic, Slovak Republic, Hungary, Slovenia, Estonia, Latvia, Lithuania, Cyprus and Malta*.

Table 2 reports currency demand data for the euro zone and the ten new EU member countries as of the end of 2002. The currency demand for the specific year represents historic seigniorage.

If demand for currency develops proportionately among the countries the relative demand for currency will be constant, and, hence, the individual countries' shares of currency represent the share of seigniorage a country brings into the monetary union. Under this assumption the shares of historic seigniorage make up a distributionally neutral key for allocating the common seigniorage among the member countries.

Table 2 also reports calculated figures for the ECB redistribution key, illustrating the structure of subscribed capital for the perceived group of euro countries in an enlarged EMU.

⁷ Estonia, Lithuania and Slovenia have adopted the ERM2 exchange rate regime from July 2004 in the aim of fulfilling the convergence criteria of demonstrating exchange rate stability in a two year period before accession to the EMU.

⁸ Denmark and U.K. are EU members with a special status. They are under no obligation to participate in the third stage of EMU, but can opt to give notice of their intention to participate. Denmark and the U.K. have given notice of non-participation in the third stage of EMU. Sweden is not a member state with a special status. Consequently it must comply with all adaptation requirements, as specified in Article 109 of the Treaty. *Source: ECB Convergence Report 2002, page 38.*

Finally, Table 2 reports figures for the ‘overpayment ratio’ calculated as the ratio between the country’s share according to the redistribution key, and the country’s share of historic seigniorage. This ratio illustrates the assessed redistribution pattern of seigniorage in case of a proportionate future growth of currency demand. A ratio above 1 points to winners and a ratio below 1 to losers.

The figures point to a substantial redistribution in favor of the new member countries from Central and Eastern Europe. These results confirm by and large the previous results of Feist (2001), taking into account the different group of perceived euro countries and different years of data.

In reality, however, the winners and losers can only be identified after the enlargement of the EMU, and what matters here is the demand for currency after the euro is adopted. For several reasons future relative demand for currency is expected to increase significantly for the new EMU member countries from Central and Eastern Europe, and the outcome of the distributional game seems still open.

Table 2: Share of national currency end year 2002 and redistribution ratio based on historic seigniorage 2002.

<i>Countries</i>	Currency	Redistribution key		Overpayment ratio
	<i>Euro millions</i>	<i>Per cent</i>		
Poland	10938	2.59	6.29	2.43
Czech Rep.	6422	1.52	1.79	1.18
Slovak Rep.	1972	0.47	0.88	1.86
Hungary	4864	1.15	1.70	1.48
Slovenia	633	0.15	0.41	2.73
Estonia	447	0.11	0.22	1.99
Latvia	935	0.22	0.36	1.66
Lithuania	1086	0.26	0.54	2.08
Cyprus	683	0.16	0.16	1.00
Malta	1068	0.25	0.08	0.32
<i>Subtotal acceding 10</i>	<i>29048</i>	<i>6.88</i>	<i>16.72</i>	<i>2.43</i>
<i>Subtotal present euro area</i>	<i>392946</i>	<i>93.12</i>	<i>83.28</i>	<i>0.89</i>
Total	421994	100	100	1.00

Source: Table 1 and International Financial Statistics, International Monetary Fund, March 2004, Time series 14A.ZF...; 99Z.ZF...; 99B.ZF... To convert national currencies to Euros we use the Euro Reference Rates, compiled by the ECB from 1999 onward, and the ECU Reference Rates compiled by the European Commission for the year 1998.

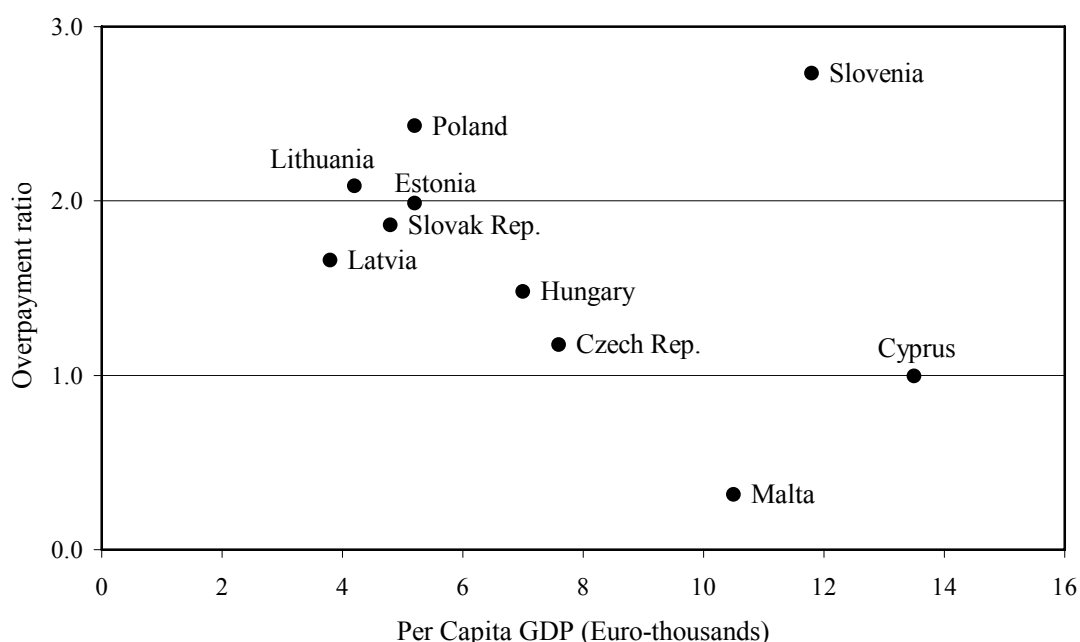
5 Some caveats

In principle the calculated ECB-shares may differ from the actual (2002) shares of currency for two reasons. Firstly, the ECB-formula may be an imprecise and flawed

approximation of the share of currency described by the demand function as specified by (1). Secondly, country specific factors may exist which influence the demand for currency in a very important way. If the latter is found to be the case, the question arises whether these specificities also hold after the country has adopted the euro. This section deals with both issues.

As shown in section 3 the ECB-formula is to the benefit of the poor countries if we, in accordance with empirical evidence, assume that $\frac{1}{2} \leq (\alpha + \gamma) \leq 1$. If, however, the redistribution ratio should reflect a systematic approximation bias the redistribution ratio should be negatively correlated with per capita income. As appears from Figure 2, there seems to be no clear relationship between the redistribution ratio and per capita income. Country specific factors seem, therefore, to be the main cause of differences between the ECB-formula and the share of currency.

Figure 2: Redistribution ratio and per capita income, 2002



Source: Table 2 and International Financial Statistics, International Monetary Fund, March 2004, Time series 99B.ZF...; 99Z.ZF.... To convert national currencies to Euros we use the Euro Reference Rates, compiled by the ECB.

There are at least two country specific factors, which may keep the shares of currency demanded in Central and East European EU-member countries at a low level before the adoption of the euro.

Firstly, the demand for currency is inversely related to the nominal interest rate. For acceding countries with a high inflation rate and high nominal interest rate the stock of currency is lower than it would have been in case that lower euro interest rates had prevailed. However, for several of the acceding countries with a hard peg or a currency board arrangement (Latvia, Estonia and Lithuania) the nominal convergence

to the euro area regarding the rate of inflation and the nominal interest rate have already taken place, yet the overpayment ratio is also high for these countries. Apparently, differences of nominal interest rates are not a major reason for the pattern of the observed redistribution ratio.

Secondly, foreign currency has, to some extent, substituted for domestic currency in the acceding countries as store of value or as parallel currency. Especially euro notes and, before 1999, German mark notes have been demanded by neighboring countries of the EU, and especially in Central and Eastern Europe and the Balkan. The reason for this diversification of currency demands is the larger credibility of foreign currency compared with domestic currency. There is no official data for the private use of foreign currency in the new member countries but the amount is far from negligible. Recently the ECB has estimated that at yearend 2003 about 9 percent of the value of the euro bank notes was in circulation by non-euro residents (ECB, 2004, p. 92). In total amount this represents about 40 billion euro, which, by comparison, is more than the total amount, about 30 billion euro, of domestically issued currency in the new member countries (see Table 2).

Although no precise data is available, part of this amount belongs to citizens in the new member countries and makes up an essential part of their currency demand. The essence of this is that the total demand for currency in the new EU-member countries from Central and Eastern Europe exceeds the stock of *domestically* issued currency or, alternatively, the demand for currency by citizen in the old euro area is less than the total stock of currency in euro⁹. Because of this partial currency substitution the ECB earns seigniorage from the new East European EU member countries already now. Using pre EMU shares of currency for distribution of post EMU seigniorage will therefore preserve this implicit taxation of the new member countries.

The contribution to seigniorage from the individual euro zone countries does not depend on the historic demand for domestic currency, but on present and future demand for euro. During the process of monetary integration, and especially in the longer term, institutional harmonization may erode country specificities. For countries, in which high inflation still prevails, convergence of nominal interest rates will take place before the change-over to the euro. This will stimulate domestic currency demand. Given the high credibility of the euro, once a country has adopted the euro, the motive for partial currency substitution will disappear. Consequently total currency demand for euro after the adoption of the euro will exceed the previous demand for domestic currency. The informal sector will probably remain relative large in the new EU-countries for several years after the euro has been adopted. This may also stimulate demand for euro, as the informal activities are more cash based, compared with registered activities.

The prospects for future economic growth and price development also point to an increase in relative demand for currency in the new member countries, compared with the present euro countries. Although there is no consensus in the various reports about

⁹ The total (domestic and foreign) currency demand is probably relative large in the new EU-member countries, because of a relative larger informal sector (shadow economy and illegal activities), see e.g. Bhattacharyya (1990) on money demand and the hidden economy and the survey article on shadow economy by Schneider and Enste (2000). This may be consistent with a relative low demand of domestic currency if currency substitution is prevalent.

the future growth for the new EU-countries, eventual convergence of living standards is expected, i.e. the rate of growth of GDP per capita is expected to be higher in the new (and poorer) member countries. Furthermore, it is widely expected that the Balassa-Samuelson effect gradually will equalize the price levels, i.e. the new member countries will experience more inflation than the present group of euro countries in the years ahead. This will also raise future currency demand in the new member countries relative to the present group of euro countries.

It should be noted that the ECB calculates the weights used for re-distribution of seigniorage only every five years. Given the specific procedure outlined in the Protocol the weights, on average, reflect four-year old data for population, and six-year old data for nominal GDP. Given the present demographic pattern of stagnant population size in the EU countries, the time lag problem for weights is related to a differential growth of nominal GDP between the new and old EU member countries.

To address this time lag let us assume that the average yearly real rate of growth of GDP in the next decades will be about 2.0 percent in the present euro countries and 4.5 percent in the new member countries. This allows for a convergent growth pattern where the gap of GDP per capita narrows at a yearly rate of 2.5 percent, assuming constant population sizes. Furthermore, let us assume that the yearly rate of inflation in the euro area is in accordance with the ECB target of 2 percent, and the rate of inflation in the new member countries is at 4.0 percent - consistent with a Balassa Samuelson effect of 2 percent. Under these assumptions nominal GDP would expand by an average yearly rate of 4 percent in the present group of euro zone countries, and 8.5 percent in the new member countries. When the ECB redistribution key is applied, its weights for the average year are based on six-year old data for nominal GDP. Let us, as a simple approximation, assume that currency demand develops proportionately to nominal GDP. Then the relative currency demand of the new fast growing member countries increases by about 25 percent during a six year period, compared with the present slow growing euro countries. Seen in this light the time-lag problem of outdated weights is far from trivial.

A retrospective view of currency demand since 1998 is reported in Figure 3 and Table 4. Figure 3 illustrates the yearly growth rates of stock of currency for the period 1999 to 2003 for the ten new member countries and the euro zone 12. To distinguish the individual countries the figure is split into two graphs illustrating the growth of currency in five countries each and the euro area.

The graphs illustrate two important conclusions. The yearly rate of growth of currency in the individual countries is very volatile, and in general the correlation of rate of growth of currency between pairs of countries is low. In other words cash flow seigniorage is very unstable¹⁰.

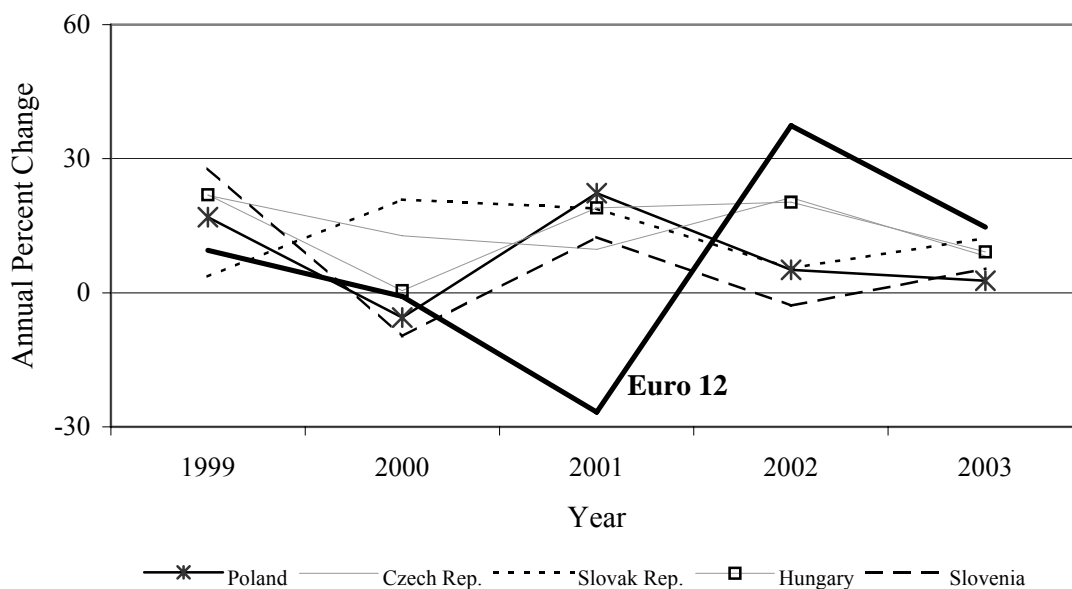
For the euro zone currency demand has also varied considerably. The physical introduction of the euro, which took place in spring of 2002, was associated with considerable contraction of currency demand during 2001, followed by a strong

¹⁰ This point has also been stressed by Gros (1993) for cash flow seigniorage for EU member countries in the ninety eighties and beginning of nineties.

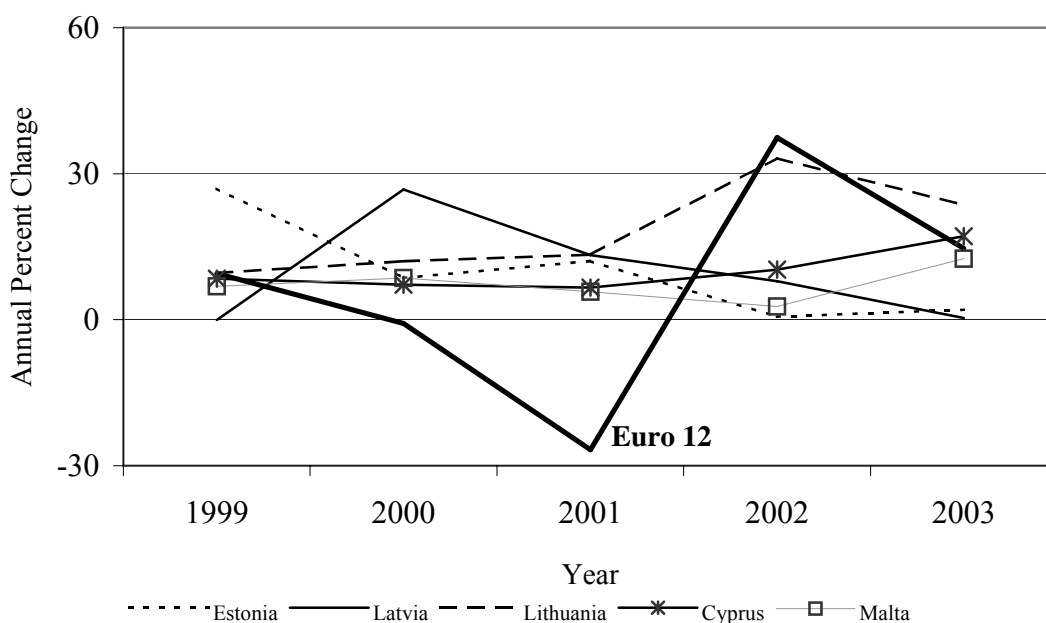
reversal of demand in 2002. From the low level of yearend 2001 currency demand for the euro12 countries subsequently expanded by about 37 percent until yearend 2002.

Figure 3: Annual rate of growth of stock of currency, 1999 - 2003

A: Poland, Czech Rep., Slovak Rep., Hungary, Slovenia and Euro 12.



B: Estonia, Latvia, Lithuania, Cyprus, Malta and Euro 12.



Source: International Financial Statistics, International Monetary Fund, March 2004.

Time series 14...ZF...; 14A..ZF...; 14C.UZW.... To convert national currencies to Euros we use the Euro Reference Rates, compiled by the ECB.

The international role of the euro is a main reason for this dramatic dip and subsequent recovery of demand for euro currency, (see ECB 2004, p.79 for this point). What may have happened is that foreigners outside the euro area gradually changed their stocks of German Mark notes and other old euro zone currencies to currencies outside the euro zone, such as US dollars, during 2001, and afterwards made a switch to euro notes in 2002 and 2003. A substantial part of this currency substitution has probably taken place in the new EU member countries. The new member countries have thus implemented a partial 'euroisation' already now - years before complete adoption of the euro.

For the period 1998 to 2003 as a whole the demand for currency in all of the new EU member countries has grown considerably faster than the growth of currency in the euro zone 12, although the opposite is the case in the last two years 2002 and 2003. This is also apparent from Table 4, which shows the share of currency for each of the new member countries and the euro zone in percent of total stock of currency, for the period 1998 to 2003. Especially Lithuania, the Czech Republic and the Slovak Republic have gained in shares of currency, and for the first two countries the gain makes up more than 50 percent of the level in 1998.

The general trend in the relative demand for currency, as well as the instability of these figures, make it problematic to assign to a given year's share of currency a prominent role for the long-term distribution key of seigniorage. The expected erosion of the present specificities in the currency demand for new member countries will increase their share of currency demand, and the initial redistribution of seigniorage wealth in favor of the new member countries from Central and Eastern Europe will recede¹¹.

¹¹ The dynamic impact of differential economic growth on formation of seigniorage is addressed briefly in Sinn and Feist (1997) as they suggest for the longer term to combine a key based on historic seigniorage with the key given by the ECB-formula. A basic amount of seigniorage should be distributed in accordance with the latest measured relative currency demand before the changeover to the euro whereas the ECB formula should be used for distribution of the future *increase* of seigniorage. However, this will clearly disfavor fast growing economies. To illustrate this assume that the monetary union consists of two groups of countries where the first group of countries stagnates and the second experiences fast growth. If the ECB formula is applied for the increase of seigniorage the stagnating countries will also capture a part of the incremental seigniorage although they have not contributed to more seigniorage.

Table 4: Stock of currency issued by acceding countries' central banks and the ECB, 1998- 2002, Euro zone 12 plus Acceding 10 = 100

<i>Countries</i>	1998	1999	2000	2001	2002	2003
	Percent of total stock of currency					
Poland	2.04	2.17	2.06	3.34	2.59	2.33
Czech Rep.	0.93	1.03	1.17	1.70	1.52	1.44
Slovak Rep.	0.33	0.31	0.38	0.60	0.47	0.46
Hungary	0.74	0.82	0.82	1.30	1.15	1.10
Slovenia	0.13	0.15	0.14	0.21	0.15	0.14
Estonia	0.08	0.09	0.10	0.14	0.11	0.09
Latvia	0.16	0.15	0.19	0.28	0.22	0.19
Lithuania	0.16	0.15	0.17	0.26	0.26	0.28
Cyprus	0.13	0.13	0.14	0.20	0.16	0.17
Malta	0.22	0.22	0.24	0.33	0.25	0.25
<i>Acceding 10</i>	<i>4.92</i>	<i>5.22</i>	<i>5.41</i>	<i>8.35</i>	<i>6.88</i>	<i>6.46</i>
<i>Euro Zone 12</i>	<i>95.08</i>	<i>94.78</i>	<i>94.59</i>	<i>91.65</i>	<i>93.12</i>	<i>93.54</i>

Source: International Financial Statistics, International Monetary Fund, March 2004.
 Time series 14...ZF...; 14A..ZF.... To convert national currencies to Euros we use the Euro Reference Rates, compiled by the ECB from 1999 onward, and the ECU Reference Rates compiled by the European Commission for the year 1998.

6 Conclusions

It is argued in this paper that the demand for currency by the new EU-member countries of Central and Eastern Europe will increase substantially, relative to the demand by the present group of euro zone countries – especially after the newcomers enter the EMU. It is therefore doubtful whether, in the long run, the application of the controversial ECB-formula will lead to big redistribution of seigniorage wealth among countries after the next enlargement of the EMU.

The problem of redistributing seigniorage in accordance with what the countries bring in of seigniorage is basically that information about the stock of circulating currency in the individual member countries ceases to exist at the very instant that the currency union is established. The currency demand in the various member countries of the EMU may only be assessed indirectly from a demand function for currency - and this approach is exactly what gives the rationale for the ECB-formula.

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Derivations

Relation (6):

Follows straight forward from inserting the nominal interest rate in equilibrium given by (5) and the assumptions about an exogenous real interest rate and an exogenous given growth rates of currency supply, population and per capita income.

Relation (7):

Using (5) and the exogenous growth rate of B gives:

$$\begin{aligned} WS_E &= \int_0^T iB e^{-it} dt \\ &= B_0 i \int_0^T e^{-(i-\mu)t} dt \\ &= B_0 \frac{-i}{(i-\mu)} \left[e^{-(i-\mu)t} \right]_0^T \\ &= B_0 \frac{(r+\mu-n-\alpha g)}{(r-n-\alpha g)} \left[1 - e^{-(r-n-\alpha g)T} \right] \\ &= B_0 \left(1 + \frac{\mu}{(r-n-\alpha g)} \right) \left[1 - e^{-(r-n-\alpha g)T} \right] \end{aligned}$$

Relation (10):

$$\begin{aligned} WS_C &= B_0 \left[1 + \mu \int_0^T e^{-(i-\mu)t} dt - e^{-(i-\mu)T} \right] \\ &= B_0 \left[1 - \frac{\mu}{i-\mu} \left[e^{-(i-\mu)t} \right]_0^T - e^{-(i-\mu)T} \right] \\ &= B_0 \left[1 - \frac{\mu}{i-\mu} e^{-(i-\mu)T} + \frac{\mu}{i-\mu} + e^{-(i-\mu)T} \right] \\ &= B_0 \left[1 + \frac{\mu}{i-\mu} \right] \left(1 - e^{-(i-\mu)T} \right) \\ &= B_0 \left[1 + \frac{\mu}{(r-n-\alpha g)} \right] \left(1 - e^{-(r-n-\alpha g)T} \right) \\ &= WS_E \end{aligned}$$